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The Pythagorean Theorem

Unit 10 Lesson 1

The Pythagorean Theorem

Students will be able to:

- Explain a proof of the Pythagorean Theorem and its converse
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

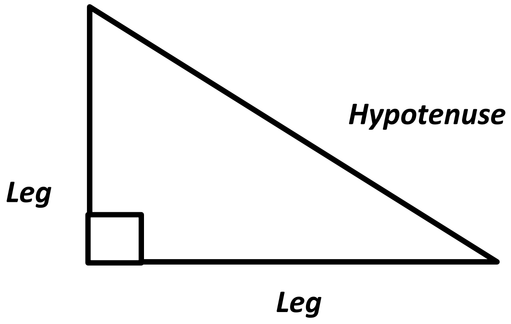
Key Vocabulary:

Right triangle, Hypotenuse, Pythagorean triple,

The Distance Formula, Square Root

The Pythagorean Theorem

- The Pythagorean Theorem describes the relationship between the lengths of the legs and the hypotenuse of a right triangle.
- In a right triangle, the side opposite the right angle is called ***the hypotenuse***. This side is always the longest side of a right triangle. The other two sides are called ***the legs*** of the triangle.



The Pythagorean Theorem

To find the length of any side of a right triangle when the lengths of the other two are known, you can use a formula by the Greek mathematician Pythagoras.

The Pythagorean Theorem

If a and b are the lengths of the legs of a right triangle, and c is the lengths of the hypotenuse, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

$$c^2 = a^2 + b^2$$

The Pythagorean Theorem

Sample Problem 1:

Find the length of the hypotenuse in the right triangle.

a. $a = 8 \qquad b = 15 \qquad c = ?$

The Pythagorean Theorem

Sample Problem 1:

Find the length of the hypotenuse in the right triangle.

a. a=8 b=15 c =?

$$c^2 = a^2 + b^2$$

$$c^2 = 8^2 + 15^2$$

$$c^2 = 64 + 225$$

$$c^2 = 289$$

$$c = \sqrt{289}$$

$$c = 17$$

The Pythagorean Theorem

Sample Problem 2:

Find the length of the missing side of the right triangle.

a. $\boldsymbol{c = 10}$ $\boldsymbol{a = 8}$ $\boldsymbol{b} = ?$

The Pythagorean Theorem

Sample Problem 2:

Find the length of the missing side of the right triangle.

$a.$ $\quad \boldsymbol{c = 10} \qquad \boldsymbol{a = 8} \qquad \boldsymbol{b =?}$

$$\boldsymbol{c^2 = a^2 + b^2}$$

$$\boldsymbol{b^2 = 10^2 - 8^2}$$

$$\boldsymbol{b^2 = 100 - 64}$$

$$\boldsymbol{b^2 = 36}$$

$$\boldsymbol{b = \sqrt{36}}$$

$$\boldsymbol{b = 6}$$

The Pythagorean Theorem

If three positive integers (a, b, c) that represent the length of each side of a right triangle, satisfy the equation

$$c^2 = a^2 + b^2$$

it is called a ***Pythagorean triple.***

The Pythagorean Theorem

Sample Problem 3: Determine whether each set of numbers form a Pythagorean triple.

$\boldsymbol{a}.\ \ (\mathbf{20}, \mathbf{21}, \mathbf{29})$

The Pythagorean Theorem

Sample Problem 3: Determine whether each set of numbers form a Pythagorean triple.

$\boldsymbol{a. (20, 21, 29)}$ $\quad \boldsymbol{a = 20} \quad \boldsymbol{b = 21} \quad \boldsymbol{c = 29}$

$$\boldsymbol{c^2 = a^2 + b^2}$$

$$\boldsymbol{29^2 = 20^2 + 21^2}$$

$$\boldsymbol{841 = 400 + 441}$$

$$\boldsymbol{841 = 841}$$

Pythagorean triple

The Pythagorean Theorem

Sample Problem 3: Determine whether each set of numbers form a Pythagorean triple.

$\boldsymbol{b}.\ (\mathbf{3}, \mathbf{6}, \mathbf{8})$

The Pythagorean Theorem

Sample Problem 3: Determine whether each set of numbers form a Pythagorean triple.

$\boldsymbol{b}.\,(\mathbf{3},\mathbf{6},\mathbf{8})$ $\qquad \boldsymbol{a}=\mathbf{3} \qquad \boldsymbol{b}=\mathbf{6} \qquad \boldsymbol{c}=\mathbf{8}$

$$\boldsymbol{c}^{2}=\boldsymbol{a}^{2}+\boldsymbol{b}^{2}$$

$$\mathbf{8}^{2}=\mathbf{3}^{2}+\mathbf{6}^{2}$$

$$\mathbf{64}=\mathbf{9}+\mathbf{36}$$

$$\mathbf{64}\neq\mathbf{45}$$

Non Pythagorean triple

The Pythagorean Theorem

- The statement than can be easily proved using a theorem is often called a **corollary.**
- The following corollary based on the Pythagorean Theorem can be used to determine whether a triangle is a right triangle.

The Pythagorean Theorem

- If a and b are measures of the shorter sides of a triangle, then c is the measure of the longest side and $c^2 = a^2 + b^2$, then the triangle is a right triangle.
- If $c^2 \neq a^2 + b^2$ then the triangle is not a right triangle.
- if $c^2 < a^2 + b^2$ then the triangle is acute, and
- if $c^2 > a^2 + b^2$ then the triangle is obtuse

The Pythagorean Theorem

Sample Problem 4:

Determine whether the following side measures form right triangle.

$\boldsymbol{a}.\ (\mathbf{4}, \mathbf{6}, \mathbf{9})$

The Pythagorean Theorem

Sample Problem 4: Determine whether the following side measures form right triangle.

$\boldsymbol{a}.\,(\mathbf{4}, \mathbf{6}, \mathbf{9}) \qquad \boldsymbol{a} = \mathbf{4} \quad \boldsymbol{b} = \mathbf{6} \quad \boldsymbol{c} = \mathbf{9}$

$$\boldsymbol{c}^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2$$

$$\mathbf{9}^2 = \mathbf{4}^2 + \mathbf{6}^2$$

$$\mathbf{81} = \mathbf{16} + \mathbf{36}$$

$$\mathbf{81} \neq \mathbf{52}$$

This is not a right triangle

The Pythagorean Theorem

Sample Problem 4: Determine whether the following side measures form right triangle.

$\mathbf{b.}\ (\mathbf{16, 30, 34})$

The Pythagorean Theorem

Sample Problem 4: Determine whether the following side measures form right triangle.

$\mathbf{b}.\ (\mathbf{16}, \mathbf{30}, \mathbf{34})$ $\quad \boldsymbol{a} = \mathbf{16}$ $\quad \boldsymbol{b} = \mathbf{30}$ $\quad \boldsymbol{c} = \mathbf{34}$

$$\boldsymbol{c}^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2$$

$$\mathbf{34}^2 = \mathbf{16}^2 + \mathbf{30}^2$$

$$\mathbf{1156} = \mathbf{256} + \mathbf{900}$$

$$\mathbf{1156} = \mathbf{1156}$$

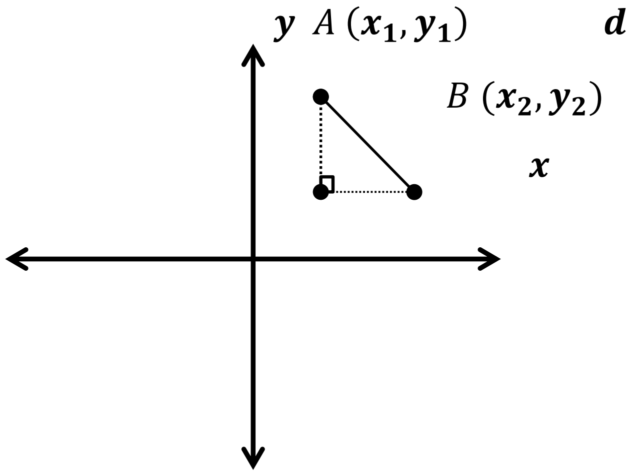
This is a right triangle

The Pythagorean Theorem

The Distance Formula

- The distance formula is derived from the Pythagorean theorem.
- To find the distance between two points (x_1, y_1) and (x_2, y_2), all that you need to do is use the coordinates of these ordered pairs and apply the formula pictured below.

The Pythagorean Theorem



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The Pythagorean Theorem

Sample Problem 5:

Find the distance between the point at $(\mathbf{2}, \mathbf{3})$ and $(-\mathbf{4}, \mathbf{6})$.

$(\boldsymbol{x_1}, \boldsymbol{y_1})$ $\qquad\qquad$ $(\boldsymbol{x_2}, \boldsymbol{y_2})$

The Pythagorean Theorem

Sample Problem 5:

Find the distance between the point at $(\mathbf{2,3})$ and $(\mathbf{-4,6})$.

(x_1, y_1) $\quad$ (x_2, y_2) $\quad$ $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(2, 3)$ $\quad$ $(-4, 6)$ $\quad$ $d = \sqrt{\left((-4) - 2\right)^2 + (6 - 3)^2}$

$d = ?$ $\quad$ $d = \sqrt{(-6)^2 + (3)^2}$

$$d = \sqrt{45}$$

$$d = 3\sqrt{5}$$